Asymmetric Nonlinear Mean Reversion in Implied Volatility

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Abstract

This study examines implied volatility for Taiwan index options contracts considering and comparing two moneyness measures, tradition and delta, using asymmetric exponential smooth transition autoregressive (AESTAR) model proposed by Sollis (2009). Our moneyness categories are divided into out-of-the-money (OTM), at-the-money (ATM), and in-the-money (ITM) groups. We test the unit root against stationary symmetric or asymmetric ESTAR nonlinearity, and then test the null of symmetric ESTAR nonlinearity against asymmetric ESTAR nonlinearity. Finally, we examine if nonlinear mean reversion in implied volatility is driven by time to maturity. Our findings show that, adjustment toward implied volatility for ATM and ITM call options considering delta moneyness measure is mean-reverting and in an asymmetric nonlinear way but symmetric nonlinear mean-reverting for ITM put options. For comparison, asymmetric nonlinear mean reversion pattern is only found for OTM and ITM call options. Finally, asymmetric nonlinear mean reversion in implied volatility for ATM call options and OTM and ATM put options with delta moneyness measure is strongly driven by different time to maturity.

Keywords: Implied volatility, asymmetric nonlinear unit root test, and mean-reverting

JEL classification: C12, C32, G12

1. Introduction

IMPLIED VOLATILITY calculated by Black-Scholes (1973) option pricing model is the unique volatility parameter for which the Bulack-Scholes formula recovers the option price. It tends to be more responsive to current market conditions than historical volatility, because it reflects the perceptions of numerous market participants about the market risk (Christensena and Prabhala, 1998; Fleming, 1998; Szakmary et al., 2003; Borovkova and Permana, 2009).[1] On the other hand, investors' uncertainty about the economic fundamentals (e.g., dividends) affects implied volatility (David and Veronesi, 2002; Guidolin and Timmerman, 2003). Therefore, understanding whether the variation in implied volatility is predictable can help us understand how underlying price movement changes over time although the empirical evidence on the predictability of implied volatility is mixed (Corrado and Miller, 2006; Mixon, 2007; Konstantinidi et al., 2008).[2]

Many financial lectures examine whether price movement for underlying assets follows random walk or mean reverting process.[3] Mean reversion in stock market prices have been extensively examined in many papers, but the empirical evidence on mean reversion in stock market prices is still inconclusive. Fama and French (1988) and Poterba and Summers (1988) document mean reversion in the U.S. stock prices and Balvers et al. (2000) report significant evidence of mean reversion in annual equity indexes for eighteen developed countries, while Lo and MacKinley (1988), Richardson and Stock (1989), and Cunado et al. (2010) yield less or no evidence against it. On the other hand, Nam et al. (2002) interpret the asymmetrical mean reversion as evidence of stock market overreaction, suggesting negative returns on average reverted more quickly, with a greater reverting magnitude, to positive returns than positive returns revert to negative returns. In the study of Bali et al. (2008), they find that the speed of mean reversion is significantly higher during the large falls of the market.

Vast research on mean reversion in price movement concentrates on the univariate properties of the series by using traditional unit root tests such as the Augmented Dickey and Fuller (ADF, Dickey and Fuller, 1979, 1981), Phillips and Perron (PP, Phillips and Perron,

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[1] It is therefore not surprising that predictions of a stock's future volatility based on implied volatilities tend to be slightly better than those based on historical volatilities.

[2] Mixon (2007) shows that the slope of at-the-money implied volatility over different maturities has predictive ability for future short-dated implied volatility, although not to the extent predicted by the expectations hypothesis.

[3] If stock price follows a mean reverting process, then there exists a tendency for the price level to return to its trend path over time and investors may be able to forecast future returns by using information on past returns. On the other hand, a random walk process says that any shock to stock price is permanent and there is no tendency for the price level to return to a trend path over time.

1988), and Kwiatkowski et al. (KPSS, 1992). On the other hand, the autoregression test introduced by Fama and French (1988) tests for mean reversion in stock prices whileas an alternative to the autoregression test of mean reversion is the variance-ratio test. However, Taylor (2001) points out that the power of the conventional ADF test is poor if the series under investigation follow a nonlinear threshold process. A non-linear exponential smooth transition autoregressive (hereafter ESTAR) model, provided by Granger and Terasvirta (1993), is useful in modeling non-linear economic relationships. Some studies have examined nonlinear mean-reversion by retaining the null hypothesis of nonstationarity against the alternative of stationary but nonlinear ESTAR processes (Taylor and Peel, 2000; Taylor et al., 2001; Kapetanios et al., 2003; McMillan, 2007; Kim, et al., 2010; Coudert et al. 2011; Sollis, 2011). Others further have examined asymmetric nonlinear mean reversion using asymmetric ESTAR (hereafter AESTAR) processes (Sollis et al., 2002; McMillan, 2007; Sollis, 2009).

Despite in the econometrics literature in the underlying index return data mean reversion of volatility is observed and captured by stochastic volatility (SV) models, few studies consider mean reversion in derivatives price movement, especially for implied volatility. Earlier empirical works take into account implied volatility in order to study mean reversion and document that implied volatility is found to be strongly mean-reverting (Merville and Piptea, 1989). Wagner and Szimayer (2004) further find the evidence of significant positive jumps in implied volatilities by estimating an autonomous mean-reverting jump-diffusion process. Recently, Wang (2007) reports that short-maturity options overreact to the dynamics of underlying assets, suggesting that mean reversion coefficients decrease in maturity. On the contrary, Stein (1989) assumes the mean reversion process for volatility and reports that long-maturity options tend to overreact to changes in the implied volatility of short-maturity options. These preceding studies only examine mean reversion in implied volatility, not nonlinear mean reversion. Specifically, little research applies the asymmetric ESTAR functional form to examine nonlinear mean reversion in implied volatility. In this paper, our findings gap the related literatures.

The Taiwan Futures Exchange (TAIFEX) introduced the Taiwan index options (TXO) in December 24, 2001. The TXO market is the third most actively traded market in Asia and the sixth most actively traded market in the world in terms of volume traded, with annual trading volume reaching 97 million contracts in 2010.[4] Moreover, the Taiwan stock and derivative markets are, due to its high liquidity and transparency, often used by international institutional investors and investment banks as a temporary surrogate for reducing or increasing exposure to other Asian markets. This particular role adds to the significance of the Taiwan stock and

[4] For detailed statistics, see the Statistics Section and Derivatives Market Survey available at the World Federation of Exchanges website, http://www.world-exchanges.org/.

derivative markets. Another motivation for our examination of the Taiwan stock and derivative markets comes from the investigation of this market in numerous prior studies. Therefore, on an examination of (a)symmetric nonlinear mean reversion in implied volatility, this paper considers Taiwan index options.

This study follows Sollis (2009) to test the unit root hypothesis against the alternative of stationary symmetric or asymmetric ESTAR nonlinearity, and then test the null of symmetric ESTAR nonlinearity against the alternative of asymmetric ESTAR nonlinearity using the AESTAR model. We examine nonlinear mean reversion in implied volatility across three moneyness categories divided into out-of-the-money (OTM), at-the-money (ATM), and in-the-money (ITM) groups with the traditional moneyness (S/K) measures and delta moneyness measure proposed by Bollen and Whaley (2004). In addition, we examine if nonlinear mean reversion in implied volatility is driven by maturities measure. Our findings shed valuable lights on nonlinear mean reversion in implied volatility. It helps us analyzing and realizing the information behind implied volatility.

Our findings show that, by considering delta moneyness measure, adjustment toward implied volatility for the ATM and ITM call options is mean-reverting and in an asymmetric nonlinear way while symmetric nonlinear mean-reverting for ITM put options. For comparison considering S/K moneyness measure, OTM and ITM call options present not only nonlinear stationery but also asymmetric ESTAR nonlinearity. To understand whether nonlinear mean reversion in implied volatility is driven by maturity, our findings show that strong adjustment toward implied volatility for ATM call options and OTM and ATM put options for the period during one day to the maturity is mean-reverting and in an asymmetric nonlinear way. On the other hand, for the period during one day to the maturity, asymmetric nonlinear mean reversion in implied volatility with delta moneyness measure is more significant than traditional moneyness measure.

Our paper is organized as follows. Section 2 introduces the econometric approaches used in this paper and describes the data. Section 3 presents the empirical results. Section 4 provides the conclusion.

2. The Nonlinear Asymmetric ESTAR Framework and Data

The nonlinear asymmetric ESTAR framework

To examine nonlinear mean reversion in implied volatility for Taiwan index options, we follow Sollis (2009) to apply an extended version of the ESTAR model allowing for symmetric or asymmetric nonlinear adjustment to test the unit root hypothesis against the alternative hypothesis of globally stationary symmetric or asymmetric ESTAR nonlinearity with a unit root. The extended ESTAR model proposed by Sollis (2009) is regarded as an asymmetric ESTAR (AESTAR) model, employing both an exponential function (G_t) and a logistic function (S_t), as follows.

$$\Delta y_t = G_t(\gamma_1, y_{t-1})\left\{S_t(\gamma_2, y_{t-1})\rho_1 + [1 - S_t(\gamma_2, y_{t-1})]\rho_2\right\} y_{t-1} + \varepsilon_t \tag{1}$$

$$G_t(\gamma_1, y_{t-1}) = 1 - \exp[-\gamma_1(y_{t-1}^2)] \qquad \gamma_1 \geq 0 \tag{2}$$

$$S_t(\gamma_2, y_{t-1}) = \{1 + \exp[-\gamma_2(y_{t-1})]\}^{-1} \qquad \gamma_2 \geq 0 \tag{3}$$

where $\varepsilon_t \sim iid(0, \sigma^2)$. Sollis (2009) indicates that global stationarity requires $\rho_1<0$, $\rho_2<0$, $\gamma_1>0$.[5] In addition, the composite function in Eq. (1) can be regarded as first-order AR parameter and is symmetric or asymmetric depending on the values of ρ_1 and ρ_2. Thus, for a particular value of $(\rho_2 - \rho_1)$, γ_2 ultimately controls the degree of asymmetry.[6] This turns out to be a useful feature of the model for deriving a test of symmetric ESTAR nonlinearity versus asymmetric ESTAR nonlinearity.

As with the original symmetric ESTAR model, the AESTAR model (Eq. (1)) can be extended to allow for higher-order dynamics, as follow.

$$\Delta y_t = G_t(\gamma_1, y_{t-1})\{S_t(\gamma_2, y_{t-1})\rho_1 + [1 - S_t(\gamma_2, y_{t-1})]\rho_2\} y_{t-1} + \sum_{j=1}^{k} \kappa_i \Delta y_{t-i} + \varepsilon_t \tag{4}$$

However, the Eq. (4) can not directly test the unit root hypothesis against the alternative hypothesis of globally stationary symmetric or asymmetric ESTAR nonlinearity with a unit root. The Eq. (4), in which transitions in the higher-order dynamic terms are not considered (Sollis et al., 2002; Kapetanios et al., 2003; Park and Shintani, 2005; Sollis, 2009), needs to transform into another equation. Therefore, the transformation of Eq. (4) is conducted by three procedures. First, assuming k=0 in Eq. (4), replacing $G_t(\gamma_1, y_{t-1})$ in Eq. (4) with a first-order Taylor expansion around γ_1=0 gives

$$\Delta y_t = \rho_1\gamma_1 y_{t-1}^3 S_t(\gamma_2, y_{t-1}) + \rho_2\gamma_1 y_{t-1}^3[1 - S_t(\gamma_2, y_{t-1})] + \eta_t \tag{5}$$

where $\eta_t = \varepsilon_t + R_t$, with R_t denoting the remainder from the Taylor expansion. Second, to simplify the model further by taking a Taylor expansion of the logistic function in Eq. (5), replacing $S_t(\gamma_2, y_{t-1})$ with $S_t^*(\gamma_2, y_{t-1}) = S_t(\gamma_2, y_{t-1}) - 0.5$ obtains $S_t^*(0, y_{t-1}) = 0$.

5 Assuming that $\gamma_1 > 0$ and $\gamma_2 \to \infty$, as y_{t-1} moves from zero toward $-\infty$ then since $S_t(\gamma_2, y_{t-1}) \to \infty$, an ESTAR transition occurs between the central regime model, $\Delta y_t = \varepsilon_t$, and the outer-regime model, $\Delta y_t = \rho_1 y_{t-1} + \varepsilon_t$, with γ_1 determining the speed of the transition. Note that Eq. (1) nests the symmetric ESTAR specification of Kapetanios et al. (2003) if $\rho_1 = \rho_2 = \rho$. On the other hand, if $\rho_1 \neq \rho_2$, the autoregressive adjustment is asymmetric either side of the attractor.

6 However, assuming $\rho1 \neq \rho2$, asymmetry can also occur for small and moderate values of γ_2, which generate a gradual transition of $S_t(\gamma_2, y_{t-1})$ between its limiting values. For $\gamma_2 \to 0$, it follows that $S_t(\gamma_2, y_{t-1}) \to 0.5$ $\forall t$, and consequently the composite function becomes symmetric irrespective of the values of ρ_1 and ρ_2.

Substituting in Eq. (5) gives

$$\Delta y_t = \rho_1^* \gamma_1 y_{t-1}^3 S_t^*(\gamma_2, y_{t-1}) + \rho_2^* \gamma_1 y_{t-1}^3 [1 - S_t^*(\gamma_2, y_{t-1})] + \eta_t \quad (6)$$

where ρ_1^* and ρ_2^* are linear functions of ρ_1 and ρ_2. Eq. (6) allows for the same pattern of nonlinearity as Eq. (5). Third, on taking a Taylor expansion of $S_t^*(\gamma_2, y_{t-1})$ in Eq. (6) around $\gamma_2 = 0$, the resulting model is

$$\Delta y_t = a(\rho_1^* - \rho_2^*)\gamma_1 \gamma_2 y_{t-1}^4 + \rho_2^* \gamma_1 y_{t-1}^3 + \eta_t \quad (7)$$

Without loss of generality, we rewrite Eq. (8) as

$$\Delta y_t = \phi_1 y_{t-1}^3 + \phi_2 y_{t-1}^4 + \sum_{j=1}^{k} \kappa_i \Delta y_{t-i} + \eta_t \quad (8)$$

where $\phi_1 = \rho_2^* \gamma_1$ and $\phi_2 = a(\rho_1^* - \rho_2^*)\gamma_1 \gamma_2$. The null hypothesis H_0: γ_1=0 in Eq. (4) becomes H_0: $\phi_1 = \phi_2 = 0$ in the AESTAR model (Eq. (8)). If the unit root hypothesis has been rejected against the alternative of stationary symmetric or asymmetric ESTAR nonlinearity, the null by hypothesis of symmetric ESTAR nonlinearity can then be tested against the alternative of asymmetric ESTAR nonlinearity using the AESTAR model (Eq. (8)) by testing H_0: $\phi_2 = 0$ against H_0: $\phi_2 \neq 0$ with a standard F-test.

Due to standard critical values cannot being used for this testing, Sollis (2009) derive the asymptotic distribution of an F-test of H_0: $\phi_1 = \phi_2 = 0$ in Eq. (8) and the test statistic is F= $(R\hat{\beta} - r)'[\hat{\sigma}^2 R\{\sum_t x_t x_t'\}^{-1} R']^{-1}(R\hat{\beta} - r)/m$.[7] Assuming k=0 in Eq. (8), it follows that $x = [y_{t-1}^3, y_{t-1}^4]'$, m=2, R is a 2×2 identity matrix, $\hat{\beta} = [\hat{\phi}_1, \hat{\phi}_2]'$ where $\hat{\phi}_1$ and $\hat{\phi}_2$ are the LS estimates of ϕ_1 and ϕ_2, r = [0, 0]′, and $\hat{\sigma}^2$ is the LS estimate of σ^2. Let F_{AE}, $F_{AE,\mu}$, $F_{AE,t}$ denote the test statistics critical values of 4.241 (2.505), 6.236 (4.557), and 8.344 (6.292) for testing H_0: ϕ_1=ϕ_2=0 for the zero mean, non-zero mean and deterministic trend cases respectively at the 1% (5%) confidence level (see Table1 in the study of Sollis, 2009).

Our Sample Data

To test the unit root hypothesis for implied volatility against the alternative of stationary symmetric or asymmetric ESTAR nonlinearity, from each observed call (C_t) or put price (P_t), we compute implied volatility σ_{it} by numerically solving the Black–Scholes call or put option pricing formula, i.e., $C_t = S_t N(d1) - K_t e^{-r_{f,t}\tau_t} N(d1 - \sigma\sqrt{\tau_t})$ or $P_t = K_t e^{-r_{f,t}\tau_t} N(-d1 + \sigma\sqrt{\tau_t}) - S_t N(-d1)$, where $d1 = [\log(S_t/K_t) + (r_{f,t} + \sigma_t^2/2)\tau_t]/\sigma_t\sqrt{\tau_t}$, τ_t denotes the time to expiration, $r_{f,t}$ stands for the interest rate, and $N(\cdot)$ denotes the standard cumulative normal distribution function.

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[7] For standard F critical values to be applicable for this test, $\phi_1 < 0$, so that under the null being tested the series is stationary. Therefore in practice such a test using standard F critical values is only asymptotically valid if the consistent LS estimate of ϕ_1 is negative.

Our sample consists of daily quotes and trades for nearby Taiwan index options (TXO) and underlying spot index. The sample period is December 24, 2001 to February 22, 2010. The sample was obtained from the Taiwan Economic Journal (TEJ). We collect the Taiwan index call or put options closing prices, strike prices, and maturities as well as the spot index closing prices. The underlying asset is the Taiwan Stock Exchange Capitalization Weighted Stock Index. The call options and put options are European style. In addition, we use the one-month time deposit interest rate in the Bank of Taiwan, as a proxy for the risk-free rate.

Prior studies show that an option's moneyness is intended to reflect its likelihood of being in the money at expiration. Typically, moneyness is measured as the relative difference between the forward price of the underlying asset and the option's exercise price, that is, S/K, hereafter as traditional moneyness measure. The greater (lower) the level of moneyness, the more likely a call (put) will be exercised at expiration. Due to moneyness affecting traders' choice among different options, in-the-money (ITM) options increase traders' trading profits (De Jong et al., 2001; Chan et al, 2009), at-the-money options (ATM) are more liquid and more sensitive to volatility, and have lower bid-ask spreads than other options (Kaul et al., 2002; Chan et al, 2009), and out-of-the-money (OTM) options play the most significant role in the price discovery process among of all options (Chakravarty et al., 2004; Chan et al, 2009). Hence, if information content of options varies with options' moneyness, pooling all options together could result in mixed findings.

Following Chan et al (2009), we only employ TXO options with strike prices between 80% and 120% of the prevailing Taiwan stock index. As options with different moneyness have distinct liquidity, leverage effect, delta (sensitivity to spot price movements), and vega (sensitivity to volatility), we examine nonlinear mean reversion in the implied volatility for index call or puts options across different ranges of options moneyness. We define OTM call (put) options as options with strike prices ranging between 102 (80)% and 120 (98)% of the underlying asset price; ATM options as options with strike prices ranging between 98% and 102% of the underlying asset price; and ITM call (put) options as options with strike prices ranging between 80 (102)% and 98 (120)% of the underlying asset price.

However, Bollen and Whaley (2004), they argue that traditional moneyness measure, the ratio of underlying price over strike price (S/K), fails to account for the fact that the likelihood that the option will be in the money at expiration also depends heavily on the volatility rate of the underlying asset and the time remaining to expiration of the option. To solve the problem, they use the option's delta where delta is sensitive to the volatility of the underlying asset as well as the option's time to expiration. In this paper, contrast to results using traditional moneyness measure, we follow Bollen and Whaley (2004) to apply the delta moneyness measure. Based on deltas proposed by Bollen and Whaley (2004), options are then placed into three moneyness categories. We define OTM call (put) options as options with absolute delta ranging between 0.125 and 0.375; ATM options as options with absolute delta ranging between 0.375 and 0.625; and ITM call (put) options as options with absolute delta ranging between 0.625 and 0.875. Note that all of the delta pairings for the calls and puts reflect the fact that buying (selling) a call and selling (buying) a put is tantamount to buying (selling) the underlying asset. A put option with a delta of −0.375 should have the same implied volatility as a call option with a delta of 0.625 by virtue of put–call parity. Options with absolute deltas below 0.125 and above 0.875 are excluded.

3. Empirical results

To provide an analytical framework for our analysis, this paper follows Sollis (2009) to test the unit root hypothesis for implied volatility against the alternative of stationary symmetric or asymmetric ESTAR nonlinearity.

Table1 Descriptive statistics of implied volatility for Taiwan Index options

MN measure	C/P	moneyness	Mean	StdDev	Max	Min	Skew
Delta	calls	OTM	0.219	0.113	0.487	0.036	0.158
		ATM	0.455	0.175	1.513	0.061	1.178
		ITM	1.340	0.586	2.000	0.102	-0.854
	puts	OTM	0.319	0.131	1.178	0.003	1.272
		ATM	0.318	0.140	1.505	0.032	1.644
		ITM	0.412	0.256	2.000	0.075	1.839
S/K	calls	OTM	0.088	0.135	0.801	0.000	1.273
		ATM	0.268	0.108	1.056	0.000	0.820
		ITM	0.637	0.316	2.000	0.000	1.091
	puts	OTM	0.316	0.126	1.241	0.131	1.538
		ATM	0.303	0.138	1.380	0.000	1.328
		ITM	0.303	0.171	1.416	0.000	0.681

Table1 reports descriptive statistics of implied volatility with different moneyness measures for the Taiwan index call or put options. For call options with delta moneyness measure, the results indicate a rather clear smirk pattern, with higher for in-the-money calls than for at- and out-of-the-money calls. By contrast, also put option volatilities are analyzed, the results indicate a rather clear U-shaped smile pattern, with the lowest average implied volatility found for the at-the-money options. For comparison, with traditional moneyness measure, the pattern is similar when considering call and put option volatility.[8]

[8] Previous studies document sizable and persistent cross-sectional differences in implied volatility. Implied

On the other hand, from this Table, it is to note that implied volatility of in-the-money call options for each moneyness measure is higher than the implied volatility of comparable calls or puts. In addition, in-the-money call or put options have higher standard deviation of implied volatilities than at- and out-of-the-money calls or puts. It suggests that investors use in-the-money options to increase traders' trading profits (De Jong et al., 2001; Chan et al, 2009). Furthermore, Calls are usually traded more frequently than puts, and this might lead to a more thorough pricing of in-the-money and out-of-the-money calls than of corresponding puts. The difference in implied volatilities between corresponding puts and calls in Table 1 is less than 1%. Our findings are consistent with Engström (2002).

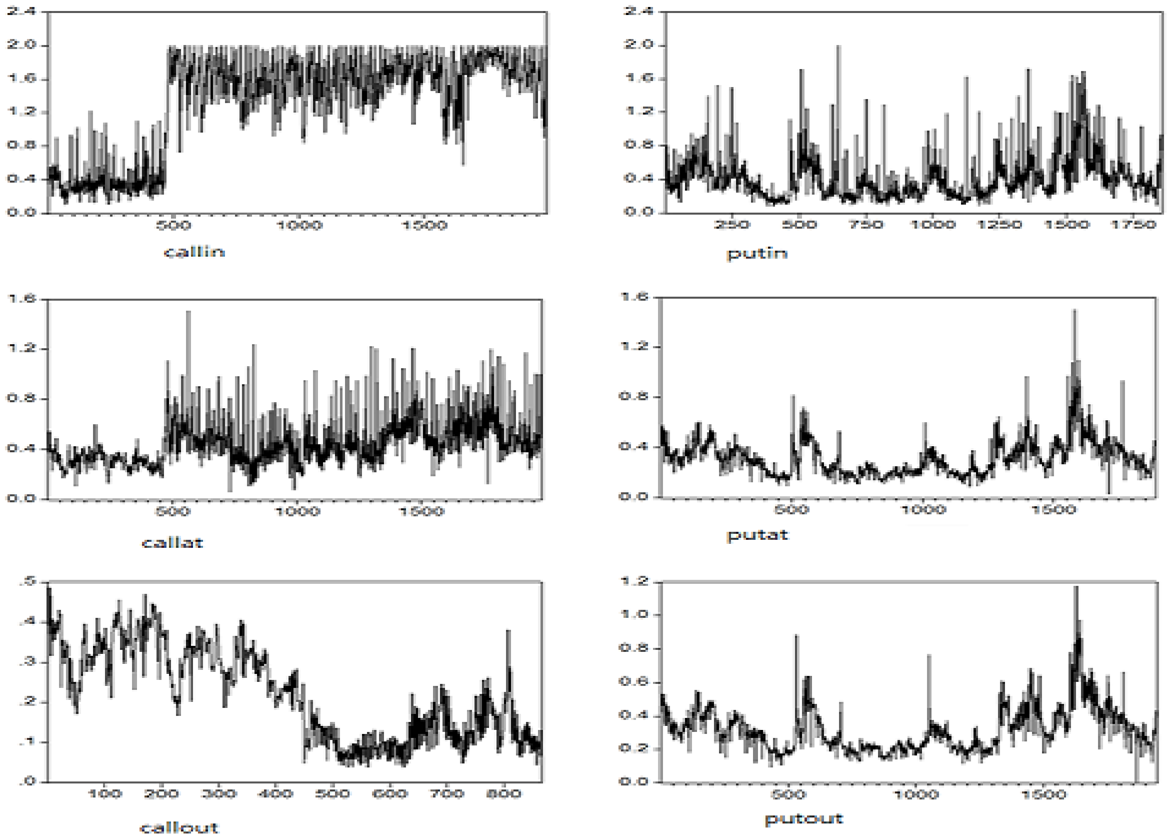


Figure1(A) The plot on implied volatility of options with delta moneyness measure

volatilities on stock and stock index options form a smile pattern prior to the October 1987 market crash where options that are deep ITM or OTM have higher implied volatilities than ATM options. After the crash, a smirk pattern appears in the stock and stock index options where the implied volatilities decrease monotonically as the exercise price increases (Dumas et al., 1998; Ederington and Guan, 2005).

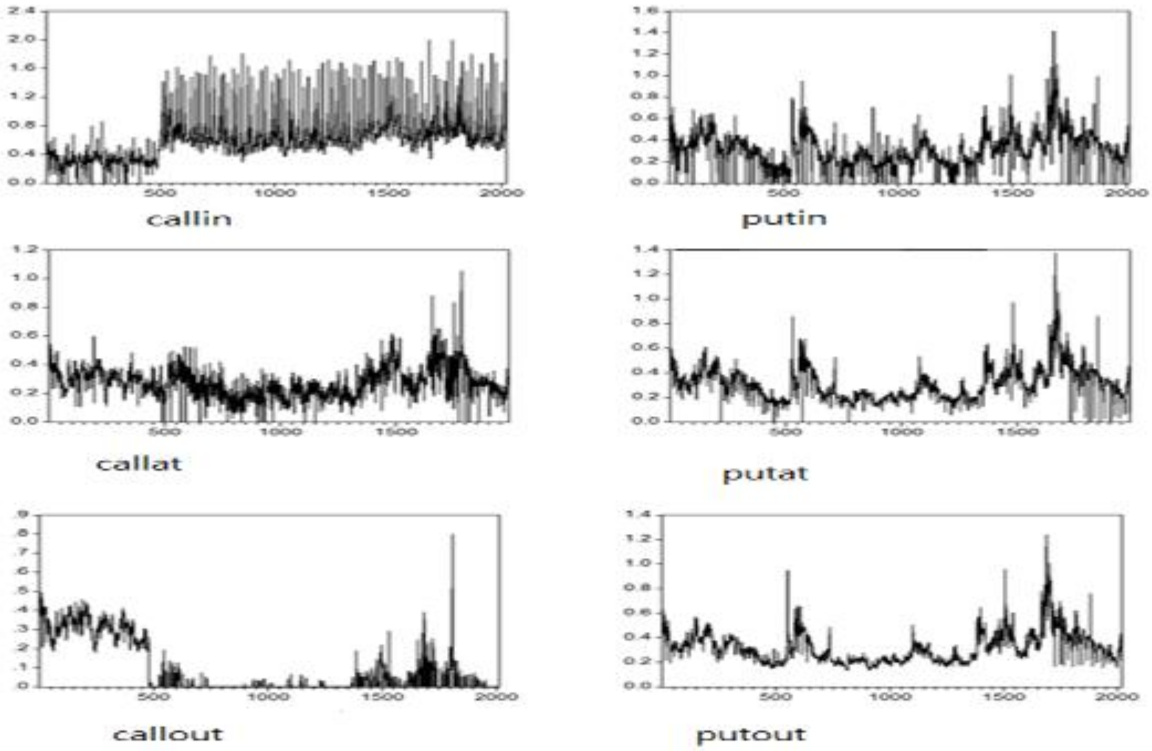


Figure1(B) The plot on implied volatility of options with traditional moneyness measure

Table2 Nonlinear unitroot tests for implied volatility based on AESTAR model

$$\Delta y_t = \phi_1 y_{t-1}^3 + \phi_2 y_{t-1}^4 + \Sigma\Delta\kappa_i y_{t-i} + \eta_t$$

MN measure	C/P	moneyness	ϕ_1	ϕ_2	H_0: $\phi_1=\phi_2=0$	H_0: $\phi_2 = 0$
Delta	calls	OTM	0.190	-0.775	2.291	0.575
		ATM	-0.003	0.295	101.507**	6.927*
		ITM	-0.089	0.053	79.729**	72.696**
	puts	OTM	-0.041	-0.007	2.172	0.014
		ATM	-0.023	-0.034	1.925	3.762
		ITM	0.024	-0.067	6.307**	0.654
S/K	calls	OTM	0.183	-0.732	18.889**	14.908**
		ATM	-0.220	0.206	2.100	0.971
		ITM	0.395	-0.349	52.421**	78.577**
	puts	OTM	-0.042	-0.018	1.983	0.088
		ATM	-0.065	0.013	1.443	0.075
		ITM	-0.091	0.013	2.664	0.039

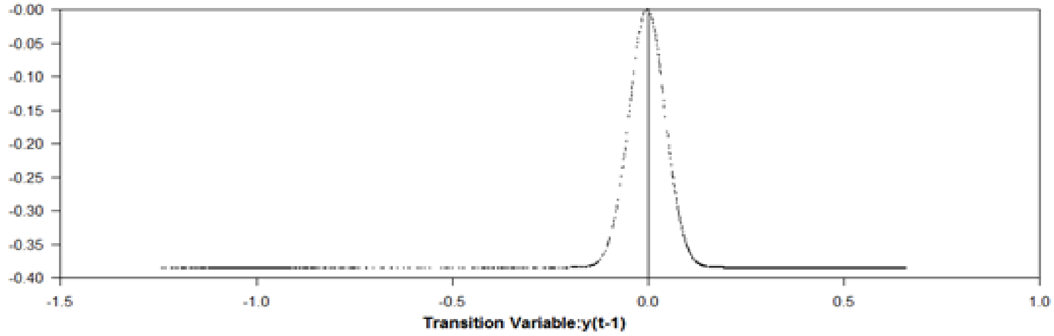
Notes: The $F_{AE,\mu}$ statistic for the null hypothesis of $\phi1 = \phi2 = 0$ are tabulated at Table 1of Sollis (2009). A feature of the AESTAR model proposed is that if the unit root hypothesis has been rejected against the alternative of stationary symmetric or asymmetric ESTAR nonlinearity, the null hypothesis of $\phi2 = 0$ (symmetric ESTAR nonlinearity) can then be tested (following a standard F distribution) against the alternative of asymmetric ESTAR nonlinearity.* and ** indicate significance at the 5% and 1% confidence level, respectively.

Figure1(A)(B) plot the series of implied volatilities on calls and puts with different moneyness considering delta and traditional moneyness measures. They seem to act nonlinearly. Note that callat (putat), callin (putin), and callout (putout) represent ATM, ITM, and OTM call (put) options, respectively.

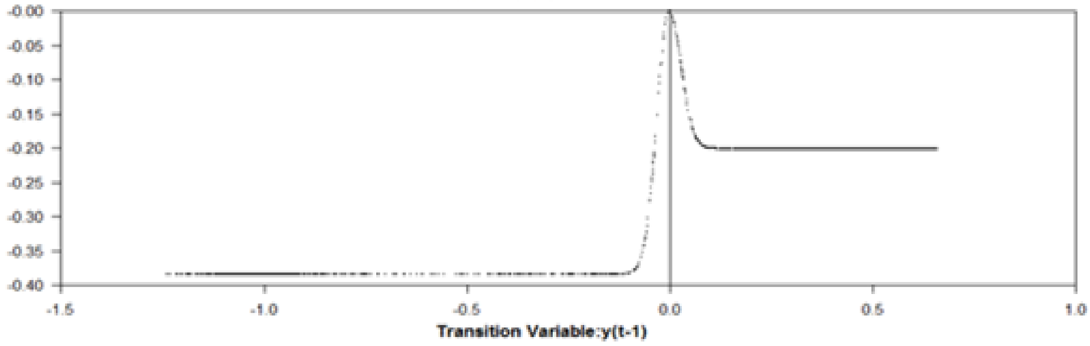
We apply AESTAR nonlinear unit root test proposed by Sollis (2009) to examine (a)symmetric patterns for implied volatility for Taiwan index calls and puts. Table2 presents results of AESTAR nonlinear unit root test. In the study of Sollis (2009), a feature of the AESTAR model proposed is that if the unit root hypothesis of $\phi_1 = \phi_2 = 0$ has been rejected against the alternative of stationary symmetric or asymmetric ESTAR nonlinearity, the null hypothesis of $\phi_2 = 0$ (symmetric ESTAR nonlinearity) can then be tested (following a standard F distribution) against the alternative of asymmetric ESTAR nonlinearity.

In Table2, to test the unit root hypothesis of $\phi_1 = \phi_2 = 0$ against the alternative of stationary symmetric or asymmetric ESTAR nonlinearity, our results present the existence of nonlinear stationery for the at-the-money and in-the-money call option as well as the in-the-money put option with delta moneyness measure. It implies the existence of nonlinear mean reversion in implied volatility on the at-the-money and in-the-money call option as well as the in-the-money put option. We further test he null hypothesis of $\phi_2 = 0$ (symmetric ESTAR nonlinearity) against the alternative of asymmetric ESTAR nonlinearity. Specifically, we find that the at-the-money and in-the-money call option exhibit asymmetric nonlinear mean reversion but the in-the-money put option exhibits symmetric nonlinear mean reversion.

For comparison considering traditional moneyness measure, our findings only show that the out-of-the-money and in-the-money call option present the existence of nonlinear stationery, suggesting mean reverting process. To further test the null hypothesis of $\phi_2 = 0$ (symmetric ESTAR nonlinearity) against the alternative of asymmetric ESTAR nonlinearity, we find that the out-of-the-money and in-the-money call option present asymmetric ESTAR nonlinearity.



(a) ESTAR model: $\Delta y_t = G_t(241.995, y_{t-1})(-0.385)$



(b) AESTAR model: $y_t = G_t(573.5, y_{t-1})\{S_t(-1074.737, y_{t-1})(-0.38) + [1 - S_t(-1074.737, y_{t-1})](-0.2)\}$

Figure2 Function plot for the in-the- money call option with delta moneyness measure: (a) ESTAR model and (b) AESTAR model

When a rejection is obtained from $F_{AE,\mu}$ it is interesting to estimate the AESTAR model in its raw form (Eq. (4)) and compare graphically with the ESTAR model in its raw form proposed by Kapetanios, et al. (2003) ($\Delta y_t = G_t(\gamma_1, y_{t-1})\rho y_{t-1} + \varepsilon_t$). We present results for the case of the in-the-money call option with delta moneyness measure. The fitted exponential function multiplied by the nonlinear AR parameter for the relevant ESTAR model, $G_t(241.995, y_{t-1})(-0.385)$, is plotted in Figure2(a) against the threshold for positive deviations from its attractor the implied volatility y_t is much more persistent than for negative deviations of the same absolute magnitude. The combined function (Eq. (9)) varies between approximately −0.2 and 0 when the implied volatility is below its attractor, but only between −0.38 and 0 when the implied volatility is above its attractor. This supports the strong rejection of symmetric ESTAR nonlinearity obtained by the second-stage test $F_{AE,\mu}$ reported in Table2. Note that the conventional ESTAR model as employed by Kapetanios et al. (2003)

and Park and Shintani (2005) do not explicitly take account of this type of asymmetric behavior.

$$\Delta y_t = G_t(573.5, y_{t-1}) \times \{S_t(-1074.737, y_{t-1})(-0.38) + [1 - S_t(-1074.737, y_{t-1})](-0.2)\} \tag{9}$$

Stein (1989) assumes the mean reversion process for volatility and reports that long-maturity options tend to overreact to changes in the implied volatility of short-maturity options but Wang (2007) indicates that short-maturity options overreact to the dynamics of underlying assets. Our findings are consistent with other findings on real exchange rate asymmetry (Sollis et al., 2002) and Stein (1989) and Wang (2007), indirect to supporting the asymmetry pattern of implied volatility.

To understand other possible explanations on implied volatility asymmetry, we examine if nonlinear mean reversion in implied volatility is driven by maturities proposed by previous studies. The results from applying the AESTAR unit root test and the second-stage test for symmetric versus asymmetric ESTAR nonlinearity are given in Table3.

Table3 Nonlinear unitroot tests for implied volatility based on AESTAR model, considering maturities

MN measure	maturity	C/P	moneyness	ϕ_1	ϕ_2	H_0: $\phi_1 = \phi_2 = 0$	H_0: $\phi_2 = 0$
Delta	1	calls	OTM	-6.922	14.784	1.811	0.009
			ATM	-0.530	0.187	164.336**	6.180**
			ITM	-0.328	0.165	3.906*	6.614*
		puts	OTM	1.264	-3.694	5387.980**	2.079
			ATM	1.294	-4.503	2990.969**	2.820
			ITM	-0.376	0.097	25.593**	1.795
	5	calls	OTM	-1.288	2.271	3.143	0.077
			ATM	-0.766	0.724	14.087**	28.002**
			ITM	-0.385	0.197	10.657**	20.666**
		puts	OTM	1.941	-5.545	12.973**	7.301**
			ATM	1.818	-5.419	43.227**	9.986**
			ITM	-0.480	0.115	2275.181**	0.751
	10	calls	OTM	0.260	-1.229	0.493	0.077
			ATM	-0.706	0.732	6.044**	12.056**
			ITM	-0.435	0.227	23.775**	47.551**
		puts	OTM	0.609	-1.588	11.964**	12.611**
			ATM	0.253	-1.192	18.088**	4.844
			ITM	-0.567	0.280	14.518**	0.082
S/K	1	Calls	OTM	1.000	-5.609	2.200	0.232
			ATM	-4.226	3.654	41.633**	19.060**
			ITM	0.234	-0.145	8.900**	17.187**

	puts	OTM	0.859	-2.771	5352.080**	6.592**
		ATM	-0.245	-2.130	793.327**	0.919
		ITM	-0.551	-0.216	6.257**	0.035
5	calls	OTM	2.226	-8.481	4.989**	1.321
		ATM	0.292	-0.512	6.923**	0.985
		ITM	0.463	-4.237	2.171	4.234
	puts	OTM	1.941	-5.513	21.336**	10.806**
		ATM	2.578	-7.669	18.953**	10.166**
		ITM	-1.482	0.728	29.982**	0.158
10	calls	OTM	1.841	-6.256	7.733**	1.107
		ATM	0.197	-1.897	8.565**	2.686
		ITM	0.362	-0.521	7.443**	11.084**
	puts	OTM	0.416	-1.351	19.371**	8.010*
		ATM	0.212	-1.099	21.127**	4.430
		ITM	-0.196	-0.629	19.493**	0.636

Notes: The $F_{AE,\mu}$ statistic for the null hypothesis of ϕ1 = ϕ2 = 0 are tabulated at Table 1of Sollis (2009). A feature of the AESTAR model proposed is that if the unit root hypothesis has been rejected against the alternative of stationary symmetric or asymmetric ESTAR nonlinearity, the null hypothesis of ϕ2 = 0 (symmetric ESTAR nonlinearity) can then be tested (following a standard F distribution) against the alternative of asymmetric ESTAR nonlinearity.* and ** indicate significance at the 5% and 1% confidence level, respectively.

Table3 reports the results from applying the AESTAR unit root test and the second-stage test for symmetric versus asymmetric ESTAR nonlinearity when examining whether nonlinear mean reversion in implied volatility is driven by maturities comparing with delta and S/K moneyness measures. We find that, on comparison of delta and S/K moneyness measure, rejections of the unit root hypothesis are obtained for ATM and ITM call as well as OTM, ATM, and ITM put options for the period during five days and ten days to the maturity at the 1% or 5% level of significance from the AESTAR test respectively while no rejections are obtained for OTM call options for the period during one day to the maturity. Except for the call for the period during one day to the maturity considering delta moneyness measure, implied volatility follows the nonlinear mean reversion process. Specifically, the ATM call option as well as the OTM and ATM put option for the period during one day to the maturity overreact to the dynamics of underlying assets as the larger magnitude of F statistics. Our findings are consistent with Wang (2007). On the other hand, the ITM call option for the period during ten days to the maturity overreacts to the dynamics of underlying assets, supporting with the argument of Stein (1989).

Furthermore, the null hypothesis of symmetric ESTAR nonlinearity is rejected against the alternative of asymmetric ESTAR nonlinearity for ATM and ITM call options across maturities. Similar results are found for OTM and ATM put options for the period during five days to the maturity for both delta and S/K moneyness measure. It suggests that adjustment toward implied volatility for the ATM and ITM call options as well as the OTM and ATM put options is mean-reverting and in an asymmetric nonlinear way. For example, for the period during one day to the maturity on comparison of delta and S/K moneyness measure, the magnitude of F statistics for ATM call options and OTM and ATM put options using delta moneyness measure are larger than S/K moneyness measure. A possible reason is that the probability of option will been exercised at the maturity day, which is related to the volatility of underlying asset and time to maturity (Bollen and Whaley, 2004). It leads to asymmetric nonlinear mean reversion in implied volatility, especial for ATM call options and the OTM and ATM put options.

4. *Conclusions*

This study follows Sollis (2009) to test the validity of implied volatility mean-reverting in sample of calls and puts across three moneyness categories comparing with the traditional moneyness (S/K) measures and delta moneyness measure proposed by Bollen and Whaley (2004). Three moneyness categories are divided into out-of-the-money (OTM), at-the-money (ATM), and in-the-money (ITM) groups. Additionally, we further examine symmetric versus asymmetric nonlinear patterns for implied volatility. Finally, we examine if nonlinear mean reversion in implied volatility is driven by maturities measure. Our sample data is the Taiwan index options (TXO) including call and put options and corresponding underlying index over December 24, 2001 to February 22, 2010.

Our findings show that, by considering delta moneyness measure, adjustment toward implied volatility for the ATM and ITM call options is mean-reverting and in an asymmetric nonlinear way while symmetric nonlinear mean-reverting for ITM put options. For comparison considering S/K moneyness measure, our findings only show that the OTM and ITM call options present the existence of nonlinear stationery. And then we further find that OTM and ITM call options present asymmetric ESTAR nonlinearity. A possible reason for asymmetry is due to the mispricing behavior on the part of investors who overreact to certain market news (Nam et al, 2001). Another possible reason is the adjustment in investors' expectation in response to predicted excess future volatility, supporting the persistence of a positive return autocorrelation (Nam et al., 2003).

To understand whether nonlinear mean reversion in implied volatility is driven by maturity, our findings show that strong adjustment toward implied volatility for the ATM call options and OTM and ATM put options for the period during one day to the maturity is mean-reverting and in an asymmetric nonlinear way. On the other hand, for the period during one day to the maturity, asymmetric nonlinear mean reversion in implied volatility with delta moneyness measure is more significant than traditional moneyness measure.

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